

New approach for alpha decay half-lives of superheavy nuclei and applicability of WKB approximation

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Abstract

The α decay half-lives of recently synthesized superheavy nuclei (SHN) are calculated by applying a new approach which estimates them with the help of their neighbors based on some simple formulas. The estimated half-life values are in very good agreement with the experimental ones, indicating the reliability of the experimental observations and measurements to a large extent as well as the predictive power of our approach. The second part of this work is to test the applicability of the Wentzel-Kramers-Brillouin (WKB) approximation for the quantum mechanical tunneling probability. We calculated the accurate barrier penetrability for alpha decay along with proton and cluster radioactivity by numerically solving Schrödinger equation. The calculated results are compared with those of the WKB method to find that WKB approximation works well for the three physically analogical decay modes.

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1 Introduction

Over the past decades, the syntheses of superheavy elements and their lifetime measurement have been explored with a variety of methods. The heavy elements with $Z = 107 - 112$ have been successfully synthesized at GSI [1]. Elements along with $Z = 113 - 116, 118$ have been produced at JINR-FLNR, Dubna [2]. Last year, two isotopes of a new element with atomic number $Z = 117$ were synthesized in the fusion reactions between ^{48}Ca projectiles and radioactive ^{249}Bk target nuclei whose α chains terminated by spontaneous fission was observed in Dubna [3], which fills the gap between the elements 116 and 118. The element 114 was independently confirmed recently by the LBNL in the USA [4] and GSI [5]. A superheavy element isotope $^{285}114$ was observed in LBNL last year [6], and an isotope of $Z = 113$ has been identified at RIKEN, Japan [7]. Thus up to now superheavy elements with $Z = 104 - 118$ have been synthesized in experiment and consequently they offer the possibility to study the heaviest known nuclei with greater detail. However, there is no consensus among theorists with regard to what should be the next doubly magic nucleus beyond ^{208}Pb . Nearly all of modern calculations predict the existence of a closed neutron shell at $N = 184$. However, they differ in predicting the atomic number of the closed proton shell. For instance, the macroscopic-microscopic model predicts the shell gap at $Z = 114$ [8–10]. The microscopic Skyrme-Hartree-Fock models give $Z = 124, 126$ [11–13] and the relativistic mean-field calculations suggest $Z = 120$ [14–16]. The magic numbers $Z = 132$ and $N = 194$ were predicted from the discontinuity of the volume integral at shell closures [17]. A tremendous progress in experiments and the development of the radioactive ion beam facilities have made it possible to reach the island of superheavy elements.

The heaviest SHN decay primarily by the emission of α -particle terminated by spontaneous fission. Therefore, in recent experiments, α decay has been indispensable for the identification of new nuclides. Because the experimentalists have to evaluate the values of the α decay half-lives, during the experimental design, it is quite important and necessary to investigate the α decay of SHN theoretically. Although α decay is very useful for the study of the nuclei, a quantitative description of them with a satisfying accuracy is difficult. The α decay was firstly interpreted as a consequence of quantum penetration of α -particle by Gamow in 1928. At present, many theoretical approaches have been being used to describe the α decay, such as the cluster model [18–20], the density-dependent M3Y (DDM3Y) effective interaction [21,22], the generalized liquid drop model (GLDM) [23–27], the Coulomb and

proximity potential model [28], the superasymmetric fission model [29, 30], the UMADAC method [31], the coupled channel approach [32, 33] and the universal curves for α and cluster radioactivities in a fission theory [34]. Some physically plausible formulas also were employed to calculate the α decay half-lives directly [29, 35–39]. Interestingly, the superasymmetric fission theory for α and cluster decay has been extended by some authors to study metallic cluster physics [40, 41], which is an example of using the nuclear methods in nanophysics.

The half-life is extremely sensitive to the α decay Q value and an uncertainty of 1 MeV in Q value corresponds to an uncertainty of α -decay half-life ranging from 10^3 to 10^5 times in the heavy element region [42]. In this work, with the experimental Q values, we carry out the half-life calculations for the recently synthesized SHN by employing a relationship between the α decay half-lives of neighboring SHN that are established based on some simple semi-empirical formulas. Differently from our approach, theoretical estimates for the lifetimes by calculating the quantum mechanical tunneling probability in a WKB framework is widely performed for the α decay along with other physically analogical decay processes. It is pointed out that the WKB approximation works well at energies well below the barrier height [43]. As a matter of fact, the accuracy of the WKB approximation also depends on the shape of the potential barrier as well as the decay energy. In this work, we obtain the penetrability with a different method and show the applicability of the WKB approximation in α , proton and cluster radioactivity.

This paper is organized as follows: In Section 2, a brief discussion of the method and the calculated results along with the corresponding discussions for the half-lives of SHN are presented. The applicability of the WKB approximation for α decay, proton and cluster emission are discussed in Section 3. Finally, a brief summary is provided in Section 4.

2 α decay half-lives of superheavy nuclei within a new approach

We start from Royer’s [44] and Viola-Seaborg semi-empirical (VSS) formulas [45, 46]. The Royer’s formula is given by

$$\log_{10} T(\text{s}) = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q}}, \quad (2.1)$$

where Q is in MeV and the parameter set varies for four types: $a = -25.31$, $b = -1.1629$, $c = 1.5864$ for even(Z)-even(N), $a = -26.65$, $b = -1.0859$, $c = 1.5848$ for even-odd, $a = -25.68$, $b = -1.1423$, $c = 1.592$ for odd-even and $a = -29.48$, $b = -1.113$, $c = 1.6971$ for odd-odd nuclei [44]. In our previous work, this formula has been extended by taking into account the centrifugal barrier to describe unfavored α decay. For odd-mass nuclei, it is possible that some decays involve non-zero l values. However, as no experimental evidence is available for the spin-parity of the levels involved in the decay, we have not included the centrifugal barrier in the present calculations. The VSS formula is given by

$$\log_{10} T(s) = (aZ + b) \frac{1}{\sqrt{Q}} + cZ + d + h_{\log}. \quad (2.2)$$

Instead of using the original set of constants by Viola and Seaborg, recent values $a = 1.64062$, $b = -8.54399$, $c = -0.19430$, $d = -33.9054$ being valid for the nuclei of four types are used [47]. h_{\log} accounts for the hindrances associated with odd proton and odd neutron numbers but does not take an effect in our calculations. Once the half-life of a nucleus $^{A_0}Z_0$ (reference nucleus) is known, the half-life of an other nucleus AZ (target nucleus) with the same type can be derived. The difference of the logarithms of half-life is written with Eq. (2.1) as

$$\begin{aligned} S &= \log_{10} T - \log_{10} T_0 \\ &= b \left(A^{1/6} \sqrt{Z} - A_0^{1/6} \sqrt{Z_0} \right) + c \left(\frac{Z}{\sqrt{Q}} - \frac{Z_0}{\sqrt{Q_0}} \right), \end{aligned} \quad (2.3)$$

and with Eq. (2.2) as

$$S = (aZ + b) \frac{1}{\sqrt{Q}} - (aZ_0 + b) \frac{1}{\sqrt{Q_0}} + c(Z - Z_0). \quad (2.4)$$

Therefore, the half-life of the nuclei AZ can be obtained from $T = 10^S T_0$ with the help of its neighboring nucleus $^{A_0}Z_0$. The two formulas can validate each other to obtain more compelling results.

Now we focus on two simple cases. One is that the two nuclei are in an isotope chain for which Eqs. (2.3) and (2.4) can be further simplified, and the other is that the two nuclei belong to an α decay chain. We estimated the α decay half-lives of recently synthesized SHN AZ with the help of the reference nuclei ^{A-2}Z and ^{A+2}Z by employing Eqs. (2.3) and (2.4) without taking into account the uncertainty of the experimental Q values. The results are presented in Table 1 compared with experimental data. The results obtained with the DDM3Y effective interaction and the GLDM are also shown for comparison. The

third column marks the experimental α decay half-lives, and the columns 4 and 5 are the estimated ones from Eq. (2.3) based on the Royer's formula and from Eq. (2.4) based on the VSS formula, respectively. The first half is obtained with the reference nuclei ^{A-2}Z and the second half are obtained with the reference nuclei ^{A+2}Z . On an average, the DDM3Y results are slightly larger than the experimental data while the GLDM values are lower than the measured ones. In fact, the DDM3Y effective interaction and GLDM are very successful because of the appropriate treatment on the microscopic level in the DDM3Y calculation and the quasimolecular shape in the GLDM in consideration of the difficulty in accurate half-life calculation. Our calculated values are in very good agreement with the experimental measurements which indicates our method is a very effective approach to investigate the half-lives of α decay when the experimental Q values are given though it is very simple in theoretical framework compared to the DDM3Y and GLDM. The two approaches based on the Royer's formula and VSS formula give nearly the same results implying the reliability of our method to a certain extent. The half-life of $^{282}113$ is underestimated by one order of magnitude with the present method and the GLDM, which are possibly due to nonzero angular momentum transfer or some nuclear structure effects such as the dramatic deformation changes as suggested in Ref. [48] and the influence of a possible neutron shell gap at $N = 166$ on its daughter nucleus. In Ref. [49], it is suggested that $N = 166$ is a neutron shell gap in certain region within relativistic mean field models. This nuclide warrants further experimental measurements with higher statistics.

Apart from calculating the decay half-life, the present method is also a useful approach to validate the experimental measurements. The recently observed SHN still await confirmation by other laboratories, which is not easy because the new SHN form an isolated island that is not linked through α decay chain with known nuclei. Therefore, the theoretical confirmations become important and necessary. Since the half-lives of the reference nuclei are taken from the experimental values, that the estimated results with Eqs. (2.3) and (2.4) are in excellent agreement with the experimental values suggests that the experimental half-lives are themselves consistent with each other which confirms the reliability of the experimental observations and measurements to a great extent.

In order to further confirm the above conclusions drawn from Table 1 and the predictive power of Eqs. (2.3) and (2.4), a relationship between the α decay half-lives of the nuclei belonging to an α decay chain is investigated here and the estimated half-lives of SHN AZ

are listed in Table 2. The columns 4 and 5 are the estimated half-lives from Eqs. (2.3) and (2.4) with the help of their daughter nuclei $^{A-4}(Z-2)$ while the columns 6 and 7 are the estimated ones with the help of their parent nuclei $^{A+4}(Z+2)$ respectively. The well agreement between the evaluated values with the experimental ones further indicates the predictive power of Eqs. (2.3) and (2.4) along with the reliability of the experimental measurements. Of course, the uncertainties in the measured values are large because of the experimental difficulties and poor statistics. And our approach underestimates the α decay half-life of $^{279}111$ as the DDM3Y and GLDM. The reason for that is just the same as that for $^{282}113$ mentioned above. Although the main shell effect has been included in the Q value to evaluate the half-life, the preformation probability is also affected obviously by the shell effect [25], which will lead to a large deviation of Eqs. (2.1) and (2.2) together with Eqs. (2.3) and (2.4) for the nuclei around the magic numbers. However, as shown in Table 2, the half-lives of element 114 and the isotones $N = 172$ are very well reproduced by applying Eqs. (2.3) and (2.4). In other words, it does not exhibit any evidence to show that $Z = 114$ and $N = 172$ are shell closures in this region.

Finally, we predict the α decay half-lives of the nuclei belonging to an α decay chain starting from $^{293}117$ without experimental values by employing Eqs. (2.3) and (2.4). They are listed in Table 3 which may be useful for future experimental measurements. The theoretical half-lives from Eq. (2.3) and Eq. (2.4) agree very well with each other which additionally confirms again the validity of our approach. The deviation in the predicted values for $^{290}115$ may be large since the experimental Q value has a large uncertainty of 0.41 MeV. The Q value for this nuclide needs to be measured with a higher accuracy. For other still unknown SHN, one can make predictions for the α decay half-lives with the decay energies calculated from the atomic mass evaluation of Audi *et al.* [50] as a substitute since the agreement with the experimental data on the mass of the known heaviest elements is very satisfactory, or from a formula for the α -decay energy [51].

3 Applicability of the WKB approximation

We turn now to the applicability of the WKB approximation for α decay. The interaction potential $V(r)$ is the sum of the nuclear potential $V_N(r)$, Coulomb potential $V_C(r)$ and the centrifugal barrier. We approximate the nuclear potential by $V_N(r) = -A_1 U_0 / [1 + \exp(\frac{r-R_0}{a})]$,

with $R_0 = 1.27A^{1/3}$ fm, $a = 0.67$ fm, $U_0 = [53 - 33(N - Z)/A]$ MeV, A_1 the mass number of the emitted particle, N, Z, A the neutron, proton, and mass numbers of the parent nucleus respectively. Here the single particle potential is taken from Ref. [52]. The Coulomb potential is given by the point-like plus uniformly charged sphere method with a parent nucleus radius $R = 1.28A^{1/3} - 0.76 + 0.8A^{-1/3}$ fm [53]. As an example, the barrier for $^{212}\text{Po} \rightarrow ^{208}\text{Pb} + \alpha$ is shown in Figure 1(a). Here only the barrier is considered and we divide the barrier into a sequence of square barriers, as shown in Figure 1(b). In principle, the barrier ranging from r_1 to infinity should be taken into account for the calculation yet it is unpractical. Therefore, we cut off the barrier at a sufficiently large distance of $r_2 = 1000$ fm and the potential barrier is divided into $n = 60000$ parts with a step of $h = (r_2 - r_1)/n$ without loss of accuracy. The wave function $u(r)$ ($\Psi(\vec{r}) = Y_{lm}(\theta, \varphi)u(r)/r$) of the emitted particle with Q value in these n regions can be written as

$$\begin{aligned}
u_1 &= A_{1,1} \exp(ik_1 r_1) + A_{2,1} \exp(-ik_1 r_1), \\
u_2 &= A_{1,2} \exp(ik_2 r_2) + A_{2,2} \exp(-ik_2 r_2), \\
&\vdots \\
u_{n-1} &= A_{1,n-1} \exp(ik_{n-1} r_{n-1}) + A_{2,n-1} \exp(-ik_{n-1} r_{n-1}), \\
u_n &= A_{1,n} \exp(ik_n r_n) + A_{2,n} \exp(-ik_n r_n),
\end{aligned} \tag{3.5}$$

with $k_j = \sqrt{2\mu(Q - V_j)/\hbar^2}$, $r_j = r_1 + (j - 1)h$ and $V_j = [V(r_j) + V(r_{j+1})]/2$. The wave function outside of the barrier is given by

$$\begin{aligned}
u_0 &= A_{1,0} \exp(ik_0 r_0) + A_{2,0} \exp(-ik_0 r_0), \\
u_{n+1} &= A_{1,n+1} \exp(ik_{n+1} r_{n+1}),
\end{aligned} \tag{3.6}$$

with $k_0 = k_{n+1} = \sqrt{2\mu Q/\hbar^2}$ where $A_{1,0}$ and $A_{1,n+1}$ are the amplitude of incident wave and transmitted wave, respectively. By using the connection condition of wave function, one can deduce the transmission amplitude and reflection amplitude for the n th square barrier

$$A_{1,n} = \frac{1}{2} \left(1 + \frac{k_{n+1}}{k_n} \right) \exp[i(k_{n+1} - k_n)r_n] A_{1,n+1}, \tag{3.7}$$

$$A_{2,n} = \frac{1}{2} \left(1 - \frac{k_{n+1}}{k_n} \right) \exp[i(k_{n+1} + k_n)r_n] A_{1,n+1}, \tag{3.8}$$

and for the j th ($j < n$) square barrier

$$A_{1,j} = \frac{1}{2} \exp(-ik_j r_j) \left[\exp(ik_{j+1} r_j) \left(1 + \frac{k_{j+1}}{k_j} \right) A_{1,j+1} + \exp(-ik_{j+1} r_j) \left(1 - \frac{k_{j+1}}{k_j} \right) A_{2,j+1} \right], \tag{3.9}$$

$$A_{2,j} = \frac{1}{2} \exp(ik_j r_j) \left[\exp(ik_{j+1} r_j) \left(1 - \frac{k_{j+1}}{k_j}\right) A_{1,j+1} + \exp(-ik_{j+1} r_j) \left(1 + \frac{k_{j+1}}{k_j}\right) A_{2,j+1} \right]. \quad (3.10)$$

The penetration probability is given by

$$P = \frac{|A_{1,n+1}|^2}{|A_{1,0}|^2}. \quad (3.11)$$

Normalization won't help-this is not a normalizable state. We choose $A_{1,n+1} = 1$, and then the $A_{1,0}$ can be recured according to the above formulas and hence one can obtain the penetrability. As a matter of fact, our method is a kind of numerical method to solve one-dimensional Schrödinger equation for unbound state, in which the differential equation is translated to recursion formulas.

Before we perform the calculation for α , proton and cluster radioactivity, we have checked this method with a soluble example

$$V(x) = V_0 \cosh^{-2}(x/a), V_0 > 0, \quad (3.12)$$

for which the exact analytic transmission probability is known [54, 55]. It is found that the result with this method completely coincides with the analytic one, which confirms the reliability of this method. Taking this method as a standard, one can test whether the WKB approximation works well or not. In the WKB approximation, the formula

$$P_{\text{WKB}} = \exp \left[-\frac{2}{\hbar} \int_{R_{\text{in}}}^{R_{\text{out}}} \sqrt{2\mu(V(r) - Q)} dr \right], \quad (3.13)$$

is employed to evaluate the penetrability, and one can estimate the relative deviation $RD = (P_{\text{WKB}} - P)/P \times 100\%$ of this WKB method. As a semiclassical approximation, there exist two classical turning points r_{in} and r_{out} in WKB method. The penetration only performs between r_{in} and r_{out} and the effects of the regions I and III of potential barrier in Figure 1(a) are neglected. However, according to quantum mechanics, the particle can be also reflected back in the region III with some probability. Additionally, the WKB method also brings some errors when one evaluates the penetrability from r_{in} to r_{out} (the region II) and it cannot deal with these Q values being near or larger than the top of the barriers in principle while our fully quantum mechanical approach has no such a drawback.

We select α decay events with $52 \leq Z \leq 118$, and the experimental Q values are taken from Refs. [2, 53]. For α decay, the RD values have been presented in Figure 2. The WKB approximation underestimates the penetration probability by about $-40\% \sim -30\%$. It

is not possible to calculate the α decay half-life theoretically with a high accuracy within the framework of barrier penetration because the preformation factor is very difficult to be estimated microscopically and the α -daughter interaction has not yet been well determined. From this point of view, the WKB approximation thus works well in investigations of α decay especially for SHN since the experimental half-lives of SHN tend to have a large uncertainty. Indeed, because the deviation is nearly a constant in the whole mass region, this constant error can be compensated by other quantities such as a phenomenological assault frequency in actual calculations within the WKB framework. In an analogous way, we investigate the deviation of the WKB approximation for proton and cluster radioactivity. The study of the nuclei far away from the β -stable line has attracted world wide attention from both the experimental and theoretical points of view. In the case of very proton-rich nuclei, it is expected to observe the proton emission experimentally [56]. Since around 1980, the cluster radioactivity was observed in experiments with daughter nuclei being almost closed-shell spherical nuclei around ^{208}Pb . The proton and cluster emission can be treated as simple quantum tunneling effects through a potential barrier just as the α decay [57, 58]. We select cluster emitters with emitted particles from ^{14}C to ^{34}Si and spherical proton emitters, for which the experimental Q values are taken from Refs. [50, 56, 59]. Figure 3(a) shows the relative deviation RD of the WKB method for the proton radioactivity of spherical proton emitters. As can be seen, the WKB method underestimates the penetrability by $-40\% \sim -20\%$ and again the deviation does not fluctuate with a large amplitude as that for α decay. Figure 3(b) presents the relative deviation RD of the WKB method for cluster radioactivity. It indicates the WKB approximation works well for the cluster emission with a deviation by only $-5\% \sim 15\%$. The RD is found to be insensitive to the nuclear potential $V_N(r)$ for these three decay modes which indicates the conclusion we draw here is universal.

4 Summary

To summarize, the α decay half-lives of newly synthesized SHN have been investigated in terms of the correlation between the half-lives of α decay. The results of the present calculations with this relationship based on the Royer's and VSS formulas are in excellent agreement with the experimental data which indicates the predictive power of our method. According to the present calculations, an important conclusion is that the experimental half-

lives are themselves consistent with each other confirming the reliability of the experimental observations and measurements to a great extent. For the nuclei $^{282}113$ and $^{279}111$, the half-lives from the theoretical estimations are underestimated by one order of magnitude possibly due to nonzero angular momentum transfer or some nuclear structure effects. The half-lives of the synthesized SHN does not exhibit any evidence to show that $Z = 114$ and $N = 172$ are shell closures in the considered region according to our analysis. The other task of the present work was to test the applicability of the WKB approximation for quantum mechanical tunneling probabilities. We calculated the barrier penetrability for α decay, proton and cluster emission accurately with the recursion formulas by dividing the potential barrier into a sequence of square barriers, which is a numerical method to solve one-dimensional Schrödinger equation for unbound state, and the results are compared with those of the WKB approximation. It is found that the WKB method produces relative deviations by about $-40\% \sim -30\%$ for α decay of heavy and superheavy nuclei, $-40\% \sim -20\%$ for proton emission and $-5\% \sim 15\%$ for cluster radioactivity. Also, in consideration of the deviations being nearly constants in each decay mode, indeed, the WKB approximation works well for these decays.

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Table 1: Calculated α -decay half-lives of recently synthesized SHN $^A Z$ within Eqs. (2.3) and (2.4) taking the nuclei in the isotope chains as references. The first eight nuclei and the rest ones are obtained with the reference nuclei ^{A-2}Z and ^{A+2}Z , respectively. The experimental data [2] and other theoretical results are also listed for comparison. Some experimental Q values are obtained by using the measured α kinetic energies taking into account the electron shielding corrections.

Nucleus	$Q^{\text{expt.}}$	$T^{\text{expt.}}$	$T^{(2.3)}$	$T^{(2.4)}$	T^{DDM3Y} [22]	T^{GLDM} [27, 44]
$^{293}_{116}$	10.67(6)	53^{+62}_{-19} ms	$62.0^{+75.8}_{-20.7}$ ms	$66.1^{+80.8}_{-22.0}$ ms	206^{+90}_{-61} ms	$22.81^{+10.22}_{-7.06}$ ms
$^{292}_{116}$	10.80(7)	18^{+16}_{-6} ms	$21.2^{+9.5}_{-5.1}$ ms	$22.7^{+10.2}_{-5.4}$ ms	39^{+20}_{-13} ms	$10.45^{+5.65}_{-3.45}$ ms
$^{289}_{115}$	10.50(9)	$0.22^{+0.26}_{-0.08}$ s	$0.13^{+0.61}_{-0.06}$ s	$0.13^{+0.65}_{-0.06}$ s	—	—
$^{289}_{114}$	9.96(6)	$2.7^{+1.4}_{-0.7}$ s	$1.6^{+0.54}_{-0.31}$ s	$1.7^{+0.58}_{-0.33}$ s	$3.8^{+1.8}_{-1.2}$ s	$0.52^{+0.25}_{-0.17}$ s
$^{284}_{113}$	10.15(6)	$0.48^{+0.58}_{-0.17}$ s	$5.6^{+10.3}_{-2.2}$ s	$4.3^{+7.9}_{-1.7}$ s	$1.55^{+0.72}_{-0.48}$ s	$0.43^{+0.21}_{-0.13}$ s
$^{285}_{112}$	9.29(6)	34^{+17}_{-9} s	$50.2^{+15.9}_{-9.3}$ s	$52.5^{+16.6}_{-9.7}$ s	75^{+41}_{-26} s	$13.22^{+7.25}_{-4.64}$ s
$^{280}_{111}$	9.87(6)	$3.6^{+4.3}_{-1.3}$ s	$2.9^{+5.2}_{-1.2}$ s	$1.9^{+3.4}_{-0.8}$ s	$1.9^{+0.9}_{-0.6}$ s	$0.69^{+0.33}_{-0.23}$ s
$^{291}_{116}$	10.89(7)	18^{+22}_{-6} ms	$15.4^{+18.0}_{-5.5}$ ms	$14.4^{+16.9}_{-5.2}$ ms	$60.4^{+30.2}_{-20.1}$ ms	$6.35^{+3.15}_{-2.08}$ ms
$^{290}_{116}$	11.00(8)	$7.1^{+3.2}_{-1.7}$ ms	$6.0^{+5.4}_{-2.0}$ ms	$5.6^{+5.0}_{-1.9}$ ms	$13.4^{+7.7}_{-5.2}$ ms	$3.47^{+1.99}_{-1.26}$ ms
$^{287}_{115}$	10.74(9)	32^{+155}_{-14} ms	$55.5^{+65.5}_{-20.2}$ ms	$52.2^{+61.7}_{-19.0}$ ms	$51.7^{+35.8}_{-22.2}$ ms	$46.0^{+33.1}_{-19.1}$ ms
$^{287}_{114}$	10.16(6)	$0.48^{+0.16}_{-0.09}$ s	$0.79^{+0.41}_{-0.21}$ s	$0.74^{+0.39}_{-0.19}$ s	$1.13^{+0.52}_{-0.40}$ s	$0.16^{+0.08}_{-0.05}$ s
$^{282}_{113}$	10.83(8)	73^{+134}_{-29} ms	$6.3^{+7.6}_{-2.2}$ ms	$8.1^{+9.8}_{-2.9}$ ms	—	$7.8^{+4.6}_{-2.8}$ ms
$^{283}_{112}$	9.67(6)	$3.8^{+1.2}_{-0.7}$ s	$2.6^{+1.3}_{-0.7}$ s	$2.5^{+1.2}_{-0.7}$ s	$5.9^{+2.9}_{-2.0}$ s	$0.95^{+0.48}_{-0.32}$ s
$^{278}_{111}$	10.89(8)	$4.2^{+7.5}_{-1.7}$ ms	$5.2^{+6.2}_{-1.9}$ ms	$8.0^{+9.6}_{-2.9}$ ms	—	$1.5^{+0.9}_{-0.5}$ ms

Table 2: Calculated α -decay half-lives of recently synthesized SHN AZ within Eqs. (2.3) and (2.4) taking the nuclei in α -decay chains as references. The results in columns 4 and 5 are obtained with the help of their daughter nuclei ${}^{A-4}(Z-2)$ while the columns 6 and 7 are the estimated ones with the help of their parent nuclei ${}^{A+4}(Z+2)$. The experimental data [2] and other theoretical results are also listed for comparison. Some experimental Q values are obtained using the measured α kinetic energies taking into account the electron shielding corrections.

Nucleus	$Q^{\text{expt.}}$	$T^{\text{expt.}}$	$T(2.3)$	$T(2.4)$	$T(2.3)$	$T(2.4)$	T^{DDM3}
${}^{294}_{118}$	11.81(6)	$0.89^{+1.07}_{-0.31}$ ms	$0.31^{+0.14}_{-0.07}$ ms	$0.32^{+0.14}_{-0.08}$ ms	—	—	0.66^{+0}_{-0}
${}^{290}_{116}$	11.00(8)	$7.1^{+3.2}_{-1.7}$ ms	—	—	$20.5^{+24.7}_{-7.2}$ ms	$19.8^{+23.8}_{-6.9}$ ms	13.4^{+7}_{-5}
${}^{293}_{116}$	10.67(6)	53^{+62}_{-19} ms	136^{+71}_{-35} ms	136^{+70}_{-35} ms	—	—	206^{+90}_{-61}
${}^{289}_{114}$	9.96(6)	$2.7^{+1.4}_{-0.7}$ s	$1.6^{+0.8}_{-0.4}$ s	$1.6^{+0.8}_{-0.4}$ s	$1.0^{+1.2}_{-0.4}$ s	$1.1^{+1.2}_{-0.4}$ s	$3.8^{+1.8}_{-1.2}$
${}^{285}_{112}$	9.29(6)	34^{+17}_{-9} s	—	—	$56.3^{+29.2}_{-14.6}$ s	$55.9^{+29.0}_{-14.5}$ s	75^{+41}_{-26} s
${}^{292}_{116}$	10.80(7)	18^{+16}_{-6} ms	$40.9^{+16.4}_{-9.2}$ ms	$43.1^{+17.2}_{-9.7}$ ms	—	—	39^{+20}_{-13} s
${}^{288}_{114}$	10.09(7)	$0.8^{+0.32}_{-0.18}$ s	—	—	$0.35^{+0.31}_{-0.12}$ s	$0.33^{+0.30}_{-0.11}$ s	0.67^{+0}_{-0}
${}^{291}_{116}$	10.89(7)	18^{+22}_{-6} ms	$23.9^{+8.0}_{-4.5}$ ms	$23.9^{+8.0}_{-4.5}$ ms	—	—	60.4^{+3}_{-2}
${}^{287}_{114}$	10.16(6)	$0.48^{+0.16}_{-0.09}$ s	$0.71^{+0.22}_{-0.13}$ s	$0.70^{+0.22}_{-0.13}$ s	$0.36^{+0.44}_{-0.12}$ s	$0.36^{+0.44}_{-0.12}$ s	1.13^{+0}_{-0}
${}^{283}_{112}$	9.67(6)	$3.8^{+1.2}_{-0.7}$ s	—	—	$2.6^{+0.9}_{-0.5}$ s	$2.6^{+0.9}_{-0.5}$ s	$5.9^{+2.9}_{-2.0}$
${}^{288}_{115}$	10.61(6)	87^{+105}_{-30} ms	116^{+140}_{-41} ms	121^{+146}_{-43} ms	—	—	410.5^{+}_{-}
${}^{284}_{113}$	10.15(6)	$0.48^{+0.58}_{-0.17}$ s	$2.8^{+3.3}_{-1.0}$ s	$2.7^{+3.2}_{-1.0}$ s	$0.36^{+0.43}_{-0.12}$ s	$0.34^{+0.42}_{-0.12}$ s	1.55^{+0}_{-0}
${}^{280}_{111}$	9.87(6)	$3.6^{+4.3}_{-1.3}$ s	$3.4^{+4.1}_{-1.2}$ s	$2.9^{+3.5}_{-1.0}$ s	$0.61^{+0.74}_{-0.22}$ s	$0.64^{+0.78}_{-0.23}$ s	$1.9^{+0.9}_{-0.6}$
${}^{276}_{109}$	9.85(6)	$0.72^{+0.87}_{-0.25}$ s	$0.36^{+0.43}_{-0.13}$ s	$0.45^{+0.53}_{-0.16}$ s	$0.75^{+0.90}_{-0.27}$ s	$0.90^{+1.08}_{-0.33}$ s	0.45^{+0}_{-0}
${}^{272}_{107}$	9.15(6)	$9.8^{+11.7}_{-3.5}$ s	—	—	$19.6^{+23.7}_{-6.8}$ s	$15.8^{+19.1}_{-5.5}$ s	10.1^{+5}_{-3}
${}^{287}_{115}$	10.74(9)	32^{+155}_{-14} ms	$22.6^{+110.6}_{-10.2}$ ms	$23.2^{+113.5}_{-10.4}$ ms	—	—	51.7^{+3}_{-2}
${}^{283}_{113}$	10.26(9)	100^{+490}_{-45} ms	$3.6^{+17.3}_{-1.7}$ s	$3.5^{+16.5}_{-1.6}$ s	141^{+684}_{-62} ms	138^{+669}_{-60} ms	201.6^{+}_{-}
${}^{279}_{111}$	10.52(16)	170^{+810}_{-80} ms	33^{+155}_{-15} ms	32^{+153}_{-15} ms	$4.7^{+22.8}_{-2.1}$ ms	$4.9^{+24.0}_{-2.2}$ ms	$9.6^{+14.}_{-5.7}$
${}^{275}_{109}$	10.48(9)	$9.7^{+46}_{-4.4}$ ms	—	—	$50.3^{+239.6}_{-23.7}$ ms	$51.0^{+243.0}_{-24.0}$ ms	2.75^{+1}_{-1}
${}^{282}_{113}$	10.83(8)	73^{+134}_{-29} ms	$29.6^{+52.8}_{-12.0}$ ms	$23.8^{+42.6}_{-9.6}$ ms	—	—	—
${}^{278}_{111}$	10.89(8)	$4.2^{+7.5}_{-1.7}$ ms	$5.5^{+10.1}_{-2.1}$ ms	$7.4^{+13.5}_{-2.8}$ ms	$10.3^{+18.9}_{-4.1}$ ms	$12.9^{+23.6}_{-5.1}$ ms	—
${}^{274}_{109}$	9.95(10)	440^{+810}_{-170} ms	832^{+3982}_{-382} ms	1122^{+5370}_{-515} ms	335^{+598}_{-135} ms	251^{+449}_{-102} ms	—
${}^{270}_{107}$	9.11(8)	61^{+292}_{-28} s	—	—	$32.1^{+59.1}_{-12.4}$ s	$23.9^{+44.0}_{-9.2}$ s	—

Table 3: Predicted α -decay half-lives of the nuclei AZ in α -decay chain starting from $^{294}117$ within Eqs. (2.3) and (2.4). The experimental Q values are obtained with the measured α kinetic energies [3] taking account of the electron shielding corrections. The results in columns 3 and 4 are obtained with their isotopes ^{A-2}Z and the ones in columns 5 and 6 are obtained with the help of $^{294}117$.

nuclei	$Q^{\text{Exp.}}(\text{MeV})$	$T^{(2.3)}$	$T^{(2.4)}$	$T^{(2.3)}$	$T^{(2.4)}$
$^{290}115$	10.14(41)	$1.9^{+2.3}_{-0.7}$ s	$1.6^{+1.9}_{-0.6}$ s	$4.8^{+22.6}_{-2.2}$ s	$3.7^{+17.7}_{-1.7}$ s
$^{286}113$	9.81(10)	$4.8^{+5.8}_{-1.7}$ s	$4.3^{+5.2}_{-1.5}$ s	$9.0^{+42.7}_{-4.2}$ s	$7.2^{+34.1}_{-3.3}$ s
$^{282}111$	9.18(10)	$9.0^{+10.8}_{-3.3}$ min	$6.6^{+7.8}_{-2.4}$ min	$2.9^{+13.8}_{-1.3}$ min	$2.0^{+9.3}_{-0.9}$ min
$^{278}109$	9.74(19)	$1.4^{+1.7}_{-0.5}$ s	$1.5^{+1.8}_{-0.5}$ s	$0.48^{+2.3}_{-0.2}$ s	$0.54^{+2.6}_{-0.3}$ s
$^{274}107$	8.98(10)	$33.2^{+39.6}_{-11.9}$ s	$32.5^{+38.8}_{-11.6}$ s	$22.4^{+106.5}_{-10.4}$ s	$19.5^{+92.5}_{-9.0}$ s

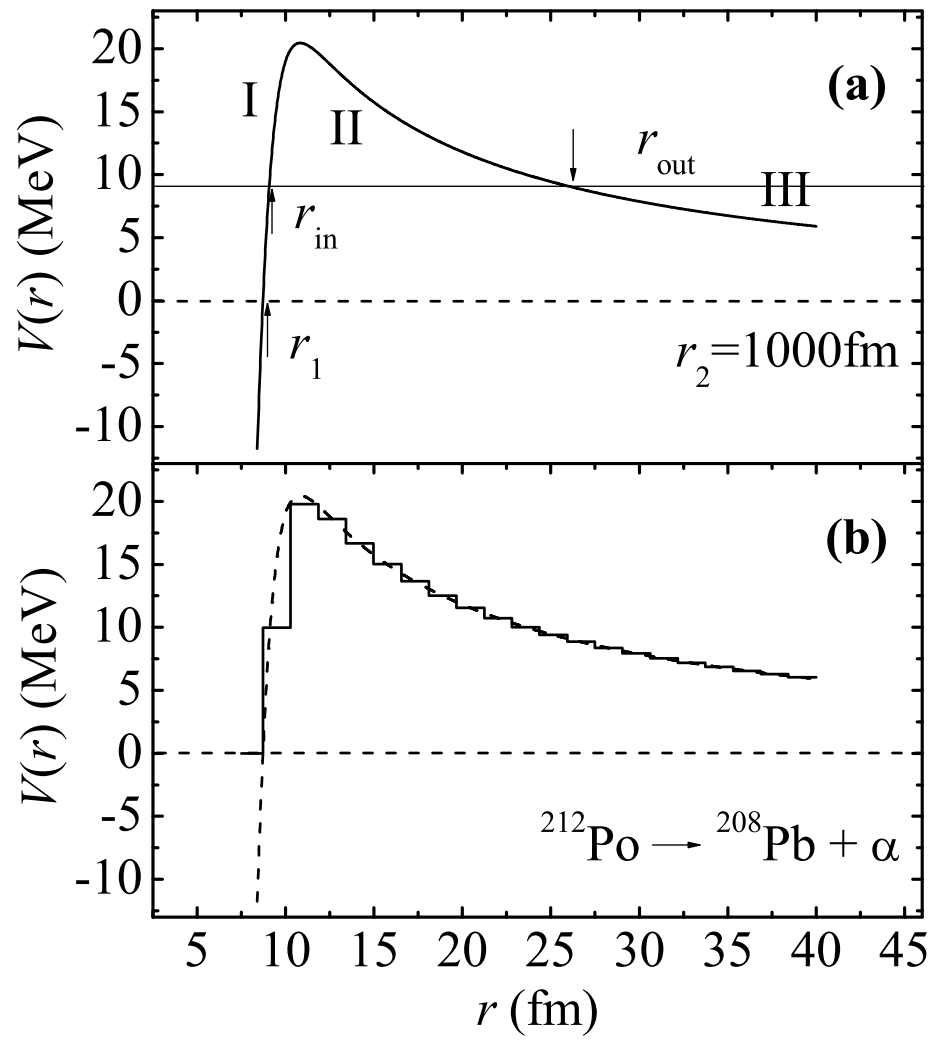


Figure 1: (a) Potential barrier of $^{212}\text{Po} \rightarrow ^{208}\text{Pb} + \alpha$. (b) It is divided into a sequence of square barriers.

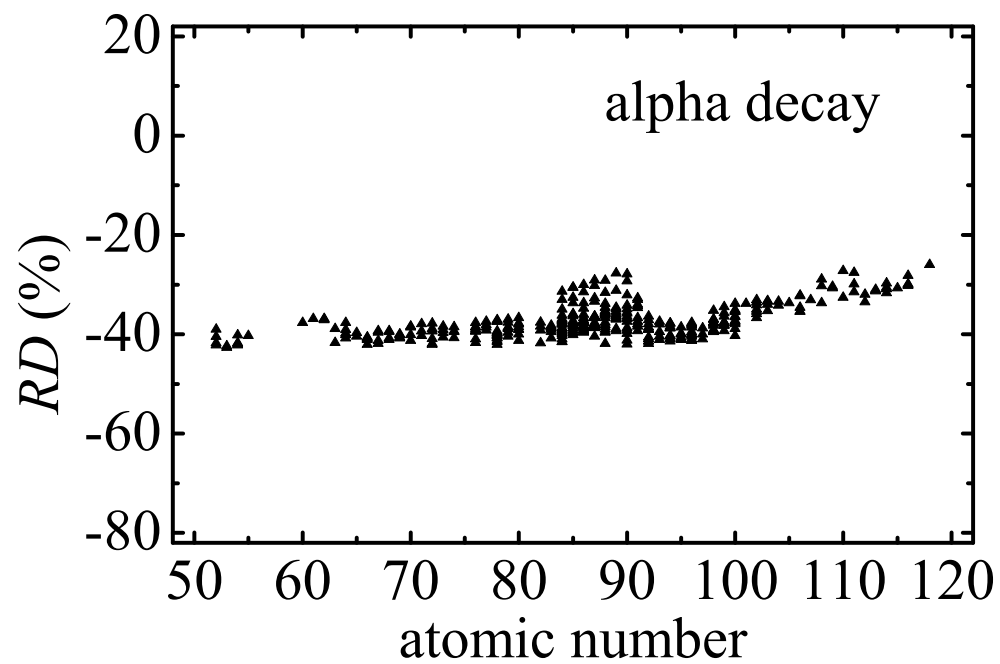


Figure 2: Relative deviation of penetrability caused by the WKB approximation for α decay.

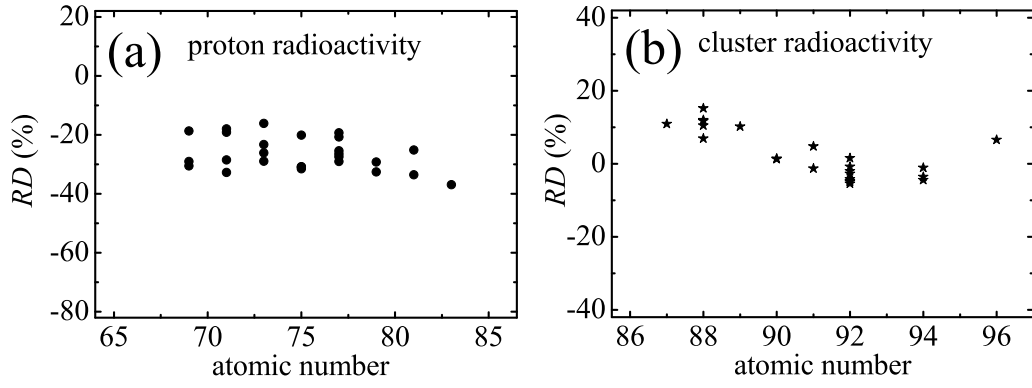


Figure 3: Relative deviation of penetrability caused by the WKB approximation for proton and cluster radioactivity.